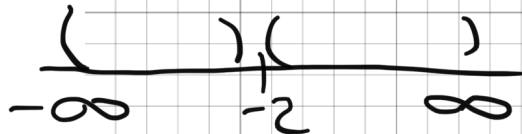Section 5.4: Properties of rational functions

1) $f(x) = \frac{2x-6}{x+2}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation $(-\infty, -2) \cup (-2, \infty)$
b) find the equation of the vertical asymptote(s) $x = -2$
c) find the equation of the horizontal asymptote $y = 2$
d) find the x- intercept $(3,0)$
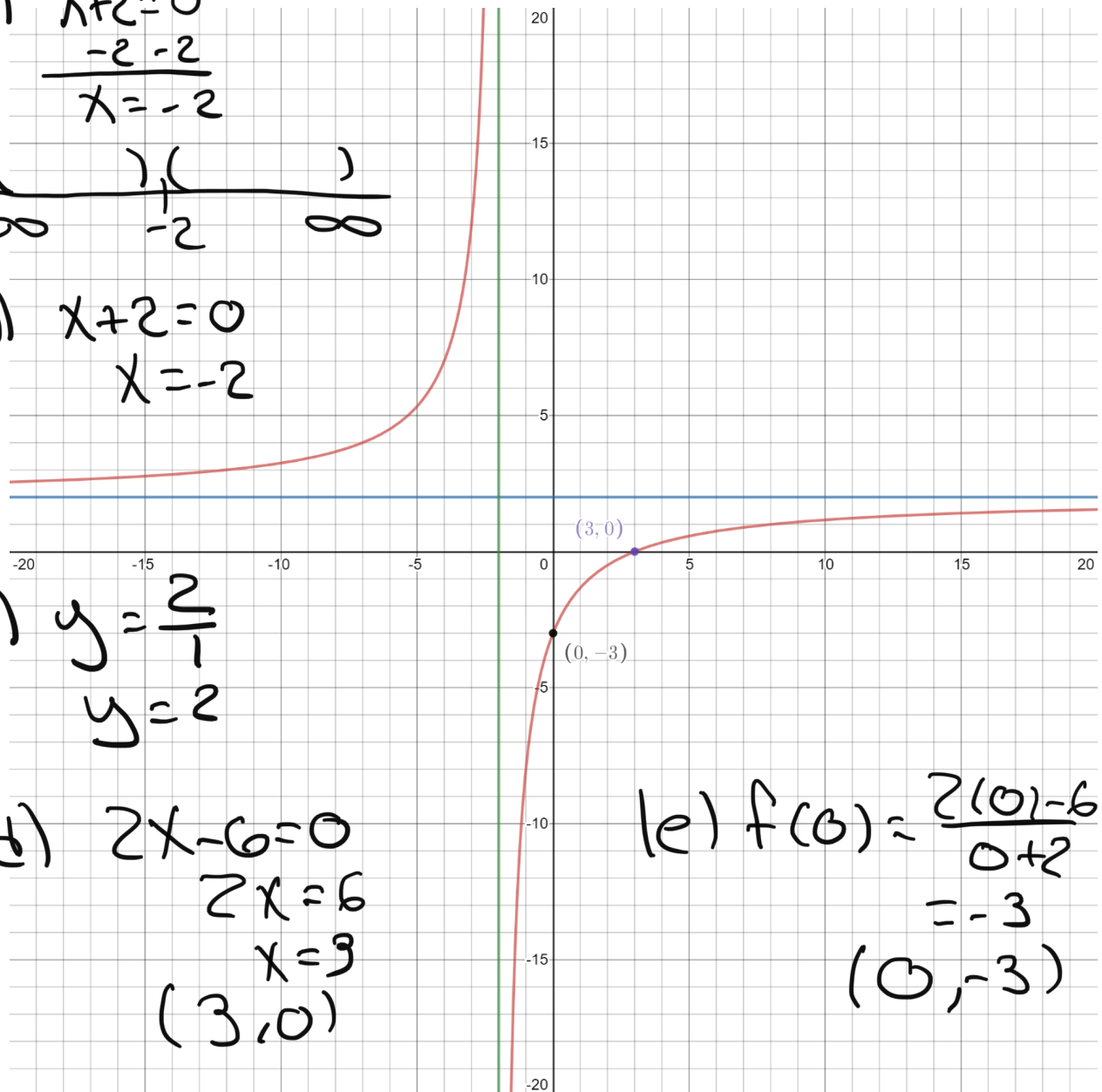e) find the y-intercept $(0, -3)$

1a) $X+2=0$

$\underline{-2\ -2}$

$X = -2$

$\underset{-\infty}{(} \quad \underset{-2}{)(} \quad \underset{\infty}{)}$

1b) $X+2=0$

$X = -2$

c) $y = \frac{2}{1}$

$y = 2$

1d) $2X-6=0$

$2X = 6$

$X = 3$

$(3,0)$

e) $f(0) = \frac{2(0)-6}{0+2}$

$= -3$

$(0, -3)$

Graph with labeled points $(3,0)$ and $(0,-3)$, vertical asymptote at $x=-2$, horizontal asymptote at $y=2$.
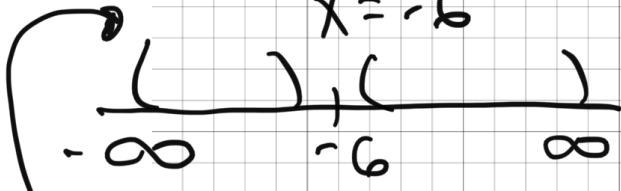
3) $f(x) = \dfrac{4x-12}{2x+12}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation $(-\infty, -6) \cup (-6, \infty)$
b) find the equation of the vertical asymptote(s) $x = -6$
c) find the equation of the horizontal asymptote $y = 2$
d) find the x- intercept $(3,0)$
e) find the y-intercept $(0. -1)$

3a) $2x+12=0$
$\quad\quad 2x=-12$
$\quad\quad\quad x=-6$

$\left(\dfrac{(\quad)_{\!,}(\quad\quad)}{-\infty \quad\quad -6 \quad\quad\quad \infty}\right)$

3b) $x=-6$

3c) $y=\dfrac{4}{2}$

$\quad\quad y=2$

3d) $4x-12=0$
$\quad\quad 4x=12$
$\quad\quad\quad x=3$
$\quad\quad (3,0)$

3e) $f(0) = \dfrac{4(0)-12}{2(0)+12} = \dfrac{-12}{12} = -1$

$\quad\quad\quad (0,-1)$
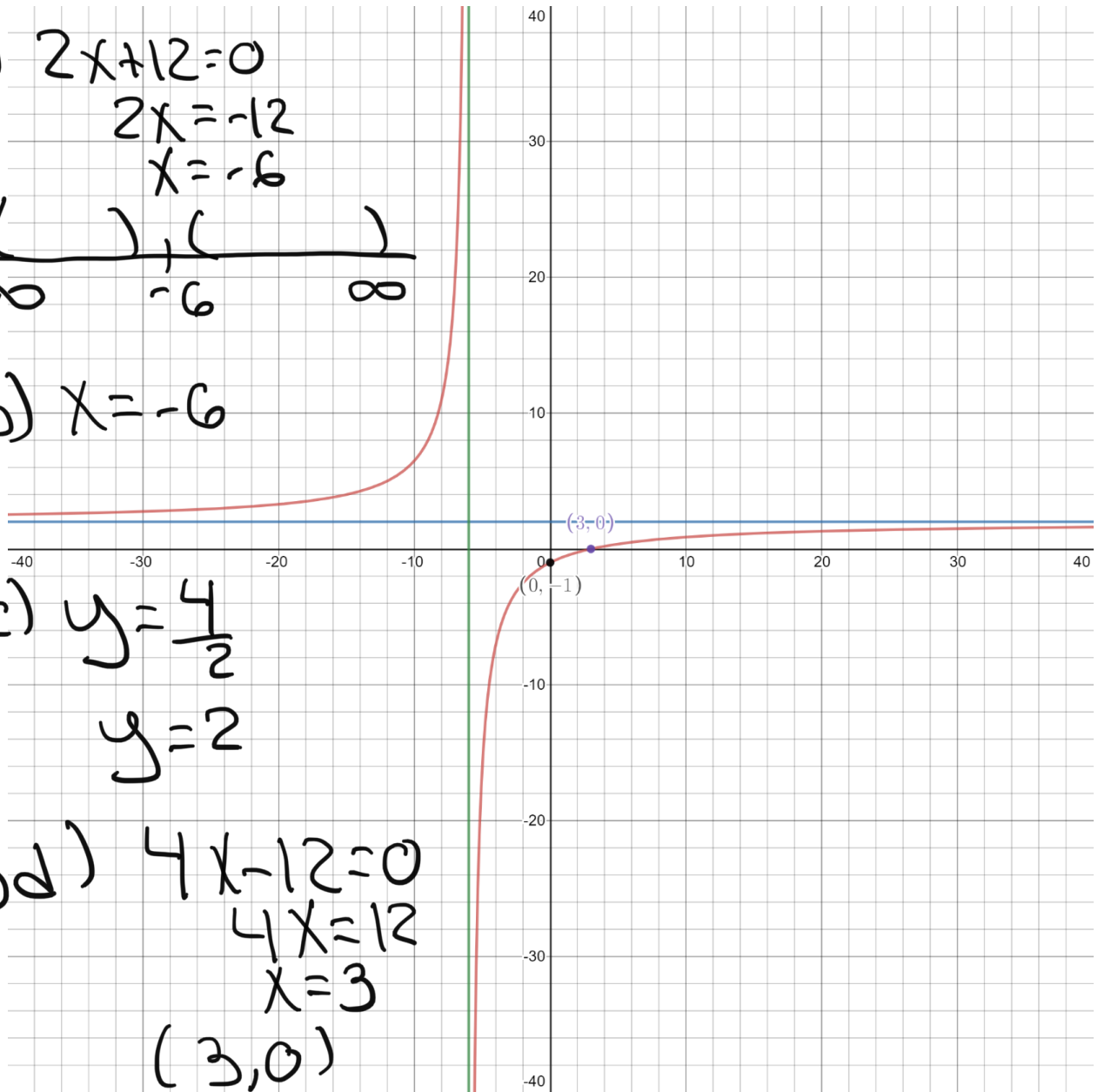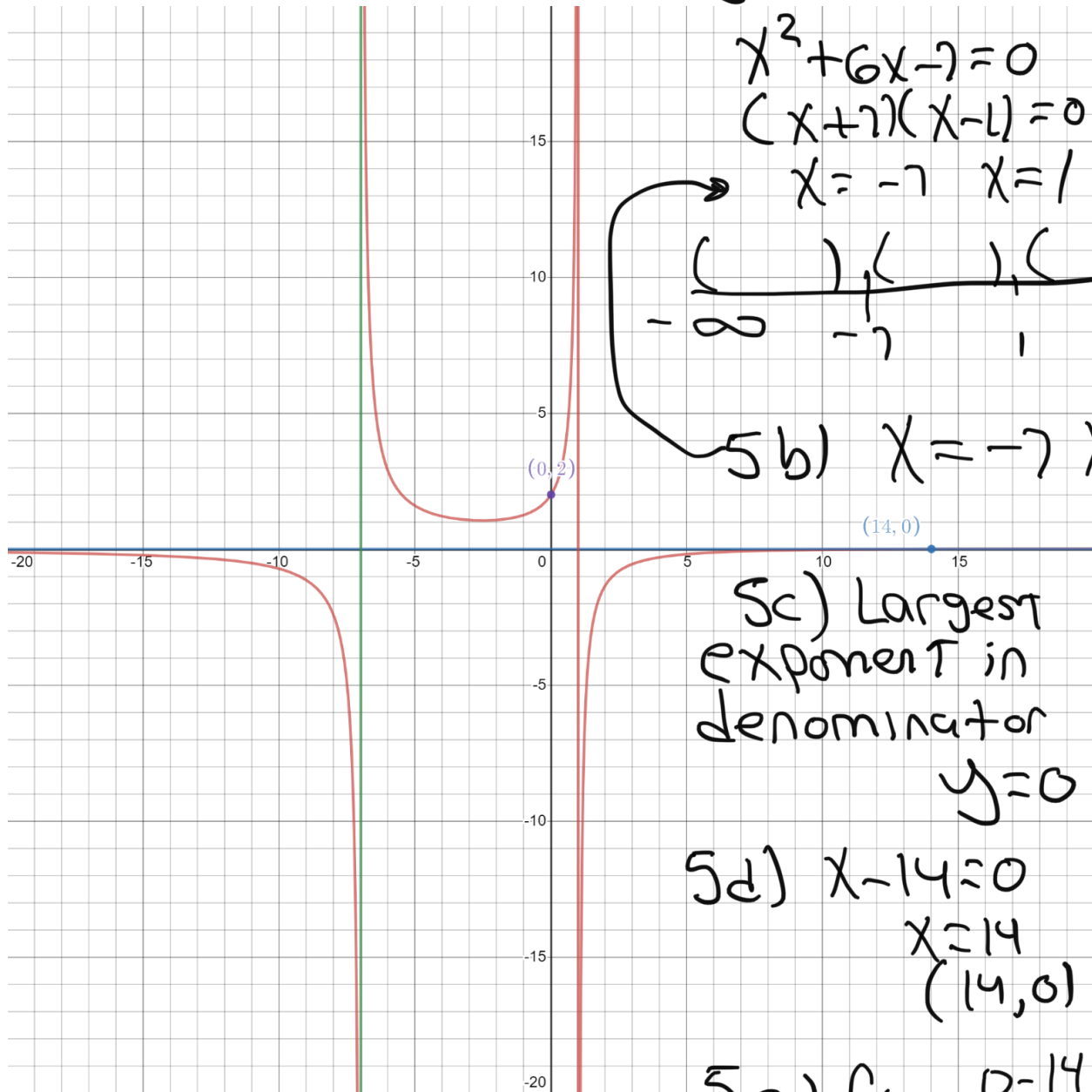
5) $f(x) = \dfrac{x-14}{x^2+6x-7}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.

a) find the domain, express your answer using interval notation $(-\infty, -7) \cup (-7, 1) \cup (1, \infty)$

b) find the equation of the vertical asymptote(s) $x = -7 \quad x = 1$

c) find the equation of the horizontal asymptote $y = 0$

d) find the x- intercept $(14, 0)$

e) find the y-intercept $(0, 2)$

5a)
$$x^2 + 6x - 7 = 0$$
$$(x+7)(x-1) = 0$$
$$x = -7 \quad x = 1$$

(sign chart)

5b) $x = -7 \quad x = 1$

(14, 0)

5c) Largest exponent in denominator
$$y = 0$$

5d) $x - 14 = 0$
$$x = 14$$
$$(14, 0)$$

5e) $f(0) = \dfrac{0 - 14}{(0)^2 + 6(0) - 7}$
$$= \dfrac{-14}{-7}$$
$$= 2$$
$$(0, 2)$$

7) $f(x) = \dfrac{x-4}{x^2 - 11x - 12}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.

a) find the domain, express your answer using interval notation $(-\infty, -1) \cup (-1, 12) \cup (12, \infty)$

b) find the equation of the vertical asymptote(s) $x = -1 \quad x = 12$

c) find the equation of the horizontal asymptote $y = 0$

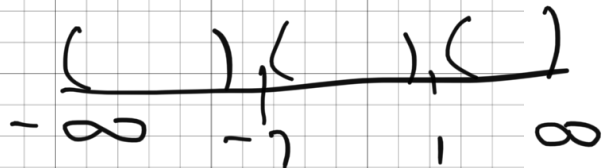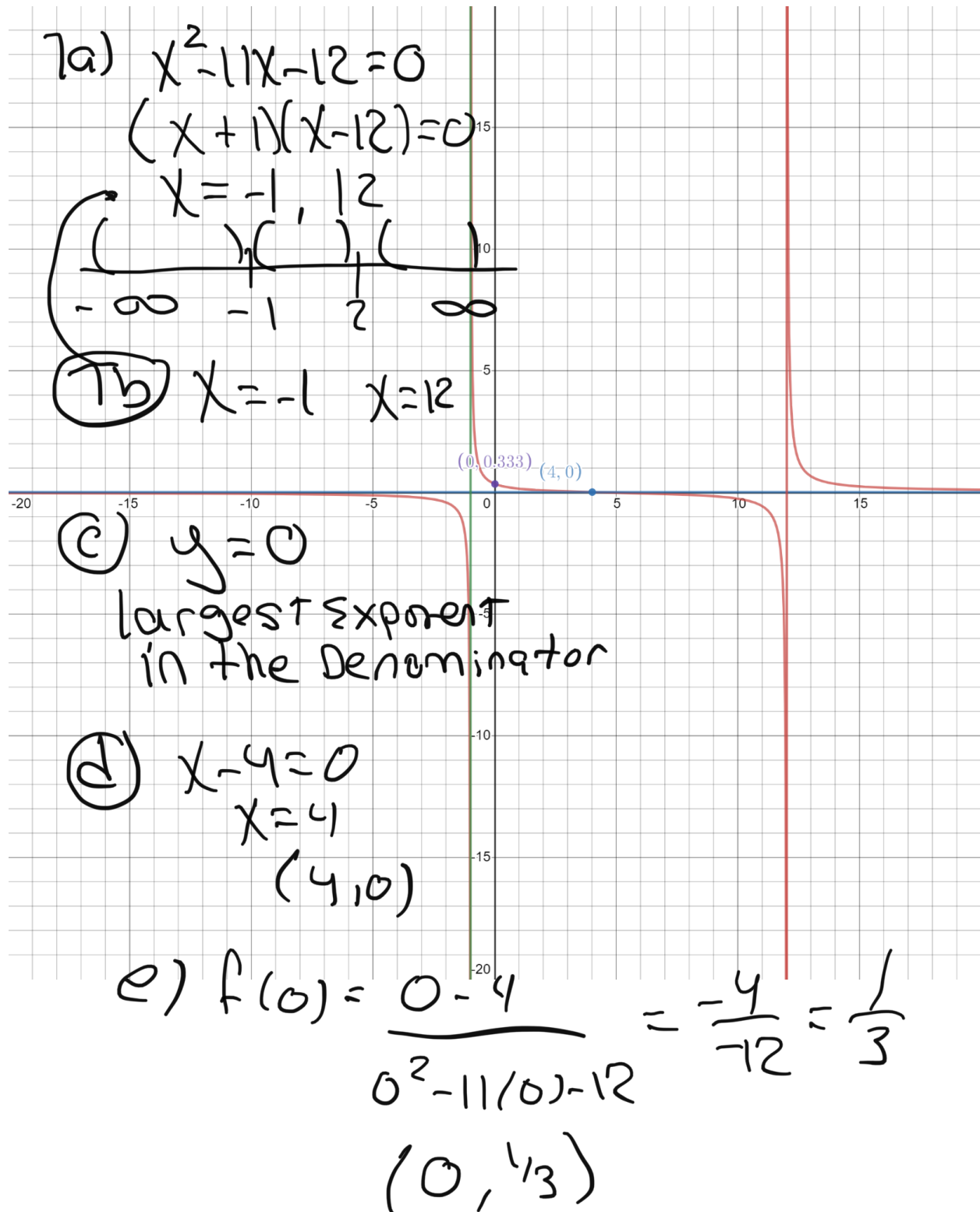d) find the x- intercept $(4, 0)$

e) find the y-intercept $\left(0, \dfrac{1}{3}\right)$

7a) $x^2 - 11x - 12 = 0$

$(x+1)(x-12) = 0$

$x = -1, \quad 12$

7b) $x = -1 \quad x = 12$

c) $y = 0$

largest exponent in the Denominator

d) $x - 4 = 0$

$x = 4$

$(4, 0)$

e) $f(0) = \dfrac{0 - 4}{0^2 - 11(0) - 12} = \dfrac{-4}{-12} = \dfrac{1}{3}$

$\left(0, \dfrac{1}{3}\right)$

9) $f(x) = \dfrac{x^2+5x-6}{x+3}$

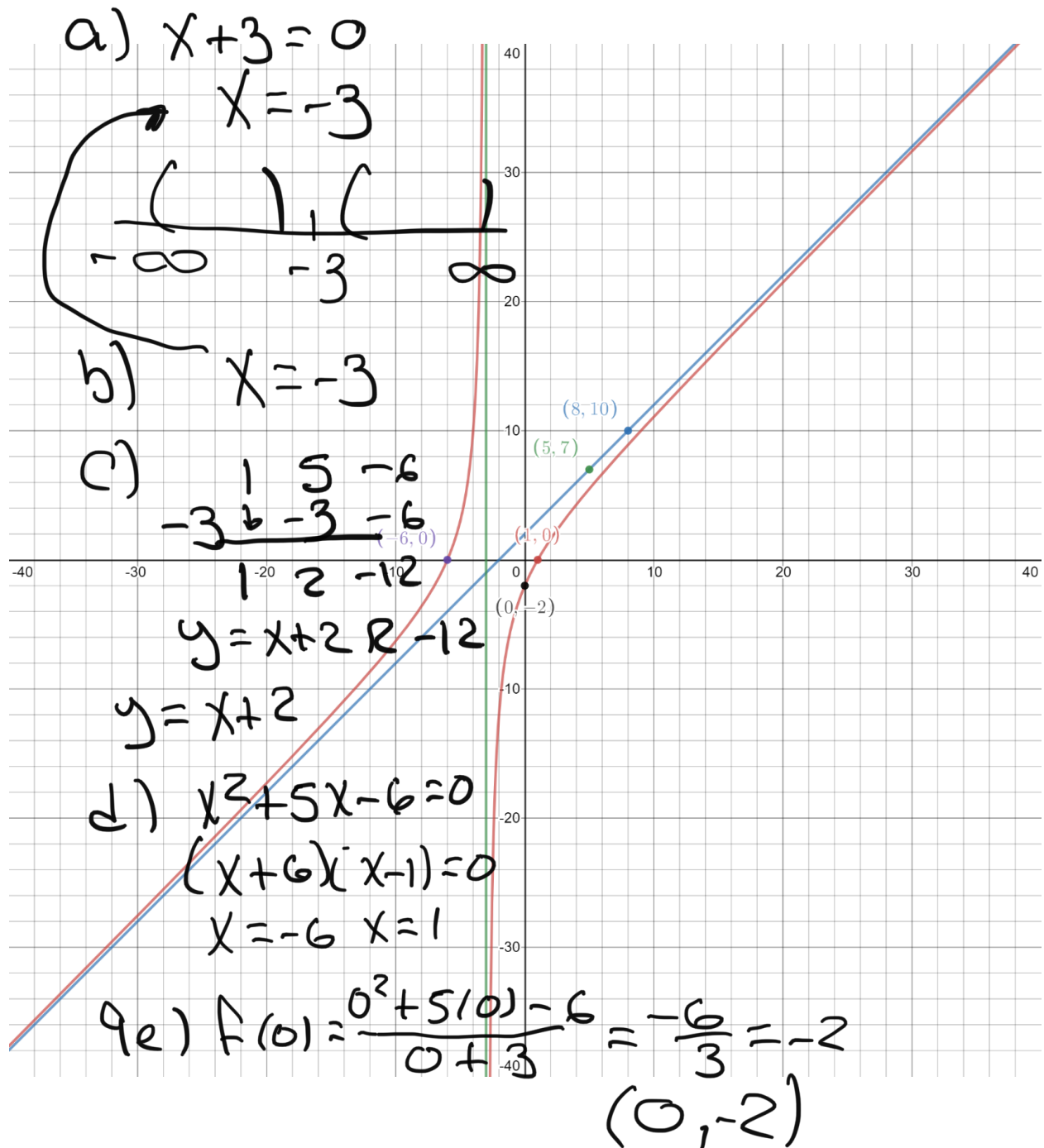Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation $(-\infty, -3) \cup (-3, \infty)$
b) find the equation of the vertical asymptote(s) $x = -3$
c) find the equation of the SLANT asymptote $y = x + 2$
d) find the x- intercept $(-6, 0)$ $(1, 0)$
e) find the y-intercept ~~(0,0)(1,0)~~ $(0, -2)$

a) $X + 3 = 0$
$X = -3$

$\overbrace{(\quad)\,(\quad)}$

$-\infty \quad -3 \quad \infty$

b) $X = -3$

c)
$$\begin{array}{r} 1 \quad 5 \quad -6 \\ -3 \underline{} \quad -3 \quad -6 \\ 1 \quad 2 \quad -12 \end{array}$$
$y = X + 2 \; R \; -12$
$y = X + 2$

d) $X^2 + 5X - 6 = 0$
$(X + 6)(X - 1) = 0$
$X = -6 \quad X = 1$

9e) $f(0) = \dfrac{0^2 + 5(0) - 6}{0 + 3} = \dfrac{-6}{3} = -2$

$(0, -2)$

(8, 10)
(5, 7)
(-6, 0)
(1, 0)
(0, -2)

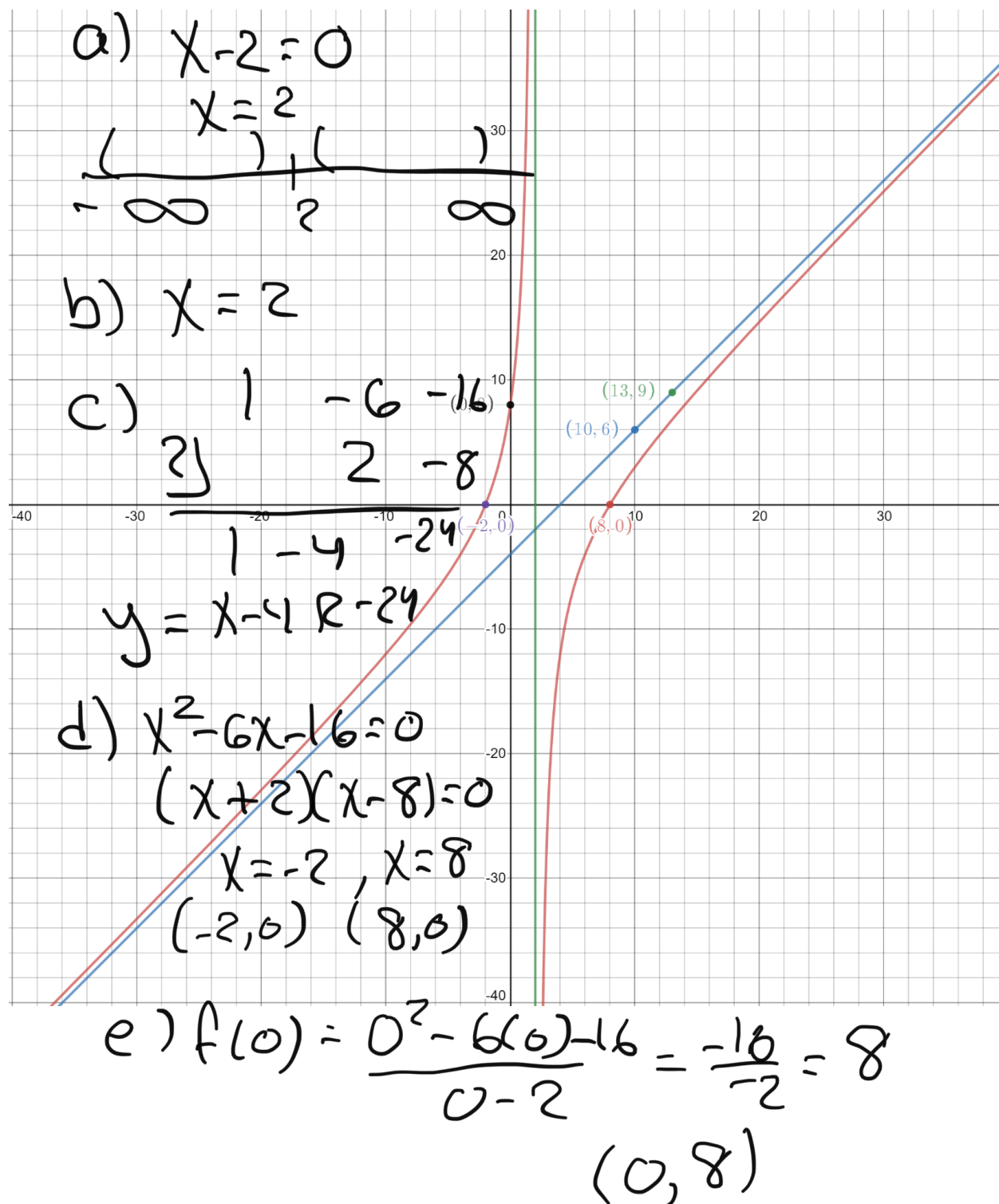11) $f(x) = \dfrac{x^2-6x-16}{x-2}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra. a) find the domain, express your answer using interval notation $(-\infty, 2) \cup (2, \infty)$
b) find the equation of the vertical asymptote(s) $x = 2$
c) find the equation of the SLANT asymptote $y = x - 4$
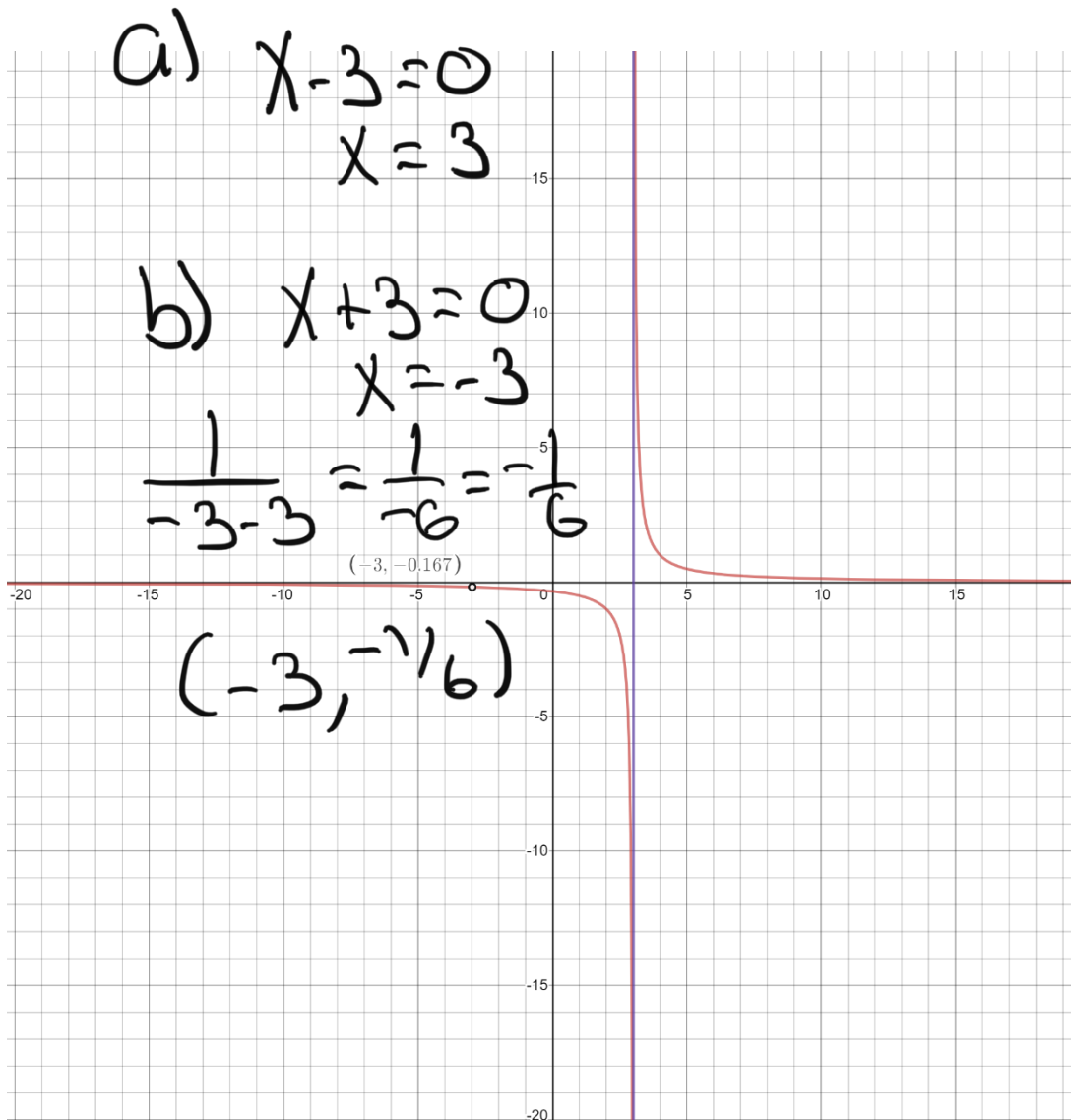d) find the x- intercept $(-2,0)$ $(8,0)$
e) find the y-intercept $(0,8)$

a) $X-2 = 0$
  $X = 2$

b) $X = 2$

c) $1 \quad -6 \quad -16$
   $2\rfloor \qquad 2 \quad -8$
   $\overline{1 \quad -4 \quad -24}$

$y = X-4 \; R-24$

d) $X^2-6X-16 = 0$
  $(X+2)(X-8) = 0$
  $X = -2, \; X = 8$
  $(-2,0) \; (8,0)$

e) $f(0) = \dfrac{0^2 - 6(0)-16}{0-2} = \dfrac{-16}{-2} = 8$

$(0,8)$

13) $f(x) = \frac{x+3}{x^2-9}$ $= \frac{x+3}{(x+3)(x-3)} = \frac{1}{x-3}$

Find the following:

$x = 3 \longrightarrow$ Hole

a) equation of the vertical asymptote $x = 3$

b) coordinates of the "hole" in the graph of $f(x)$ $\left(-3, -\frac{1}{6}\right)$

a) $x-3=0$
$x=3$

b) $x+3=0$
$x=-3$

$\frac{1}{-3-3} = \frac{1}{-6} = -\frac{1}{6}$

$(-3, -0.167)$

$(-3, -\frac{1}{6})$

15) $f(x) = \dfrac{x+2}{x^2-6x-16}$ $= \dfrac{x+2}{(x+2)(x-8)} \approx \dfrac{1}{x-8}$

↑ hole

Find the following:

a) equation of the vertical asymptote $x = 8$

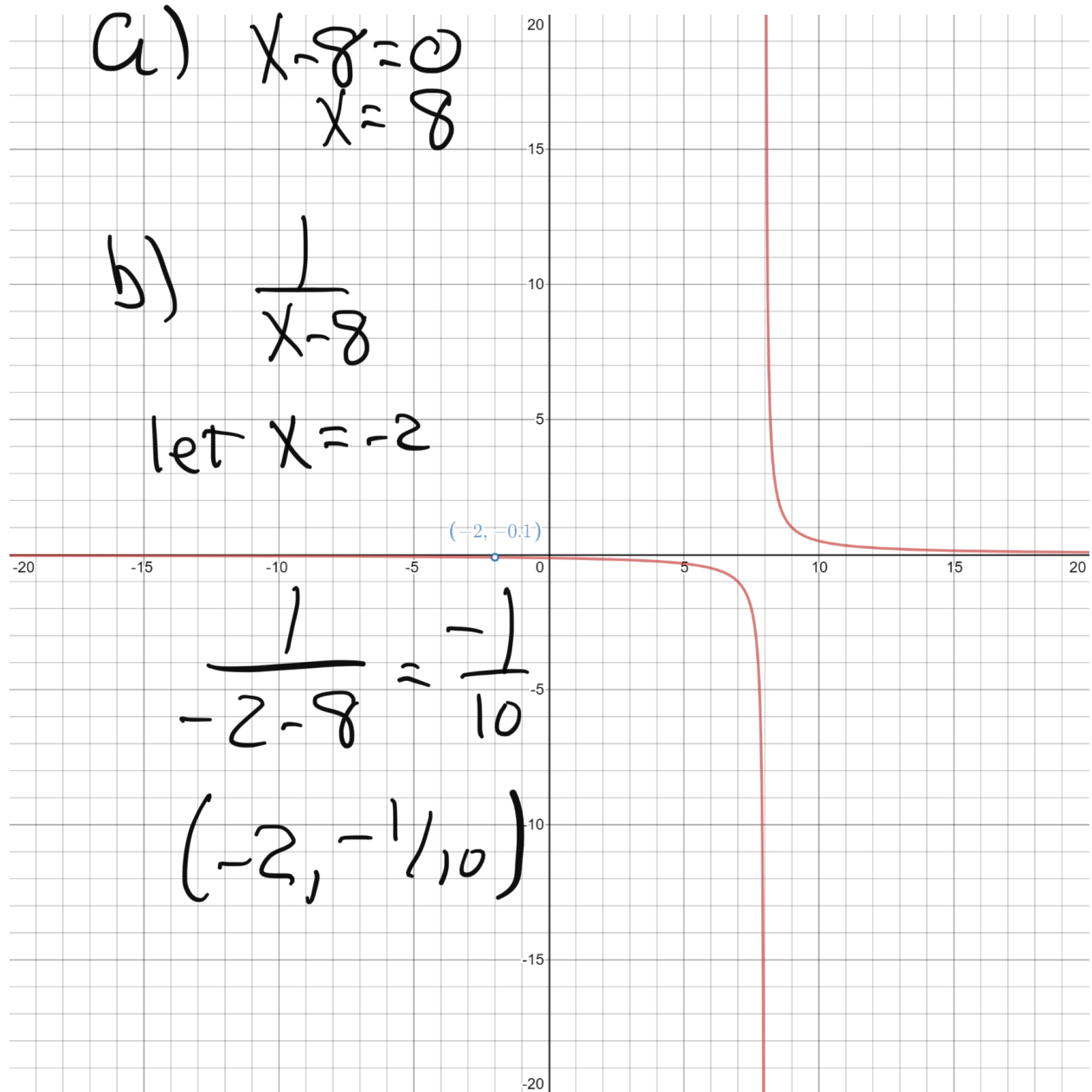b) coordinates of the "hole" in the graph of $f(x)$ $\left(-2, -\dfrac{1}{10}\right)$

a) $x - 8 = 0$
   $x = 8$

b) $\dfrac{1}{x-8}$

let $x = -2$

$\dfrac{1}{-2-8} = \dfrac{-1}{10} = -5$
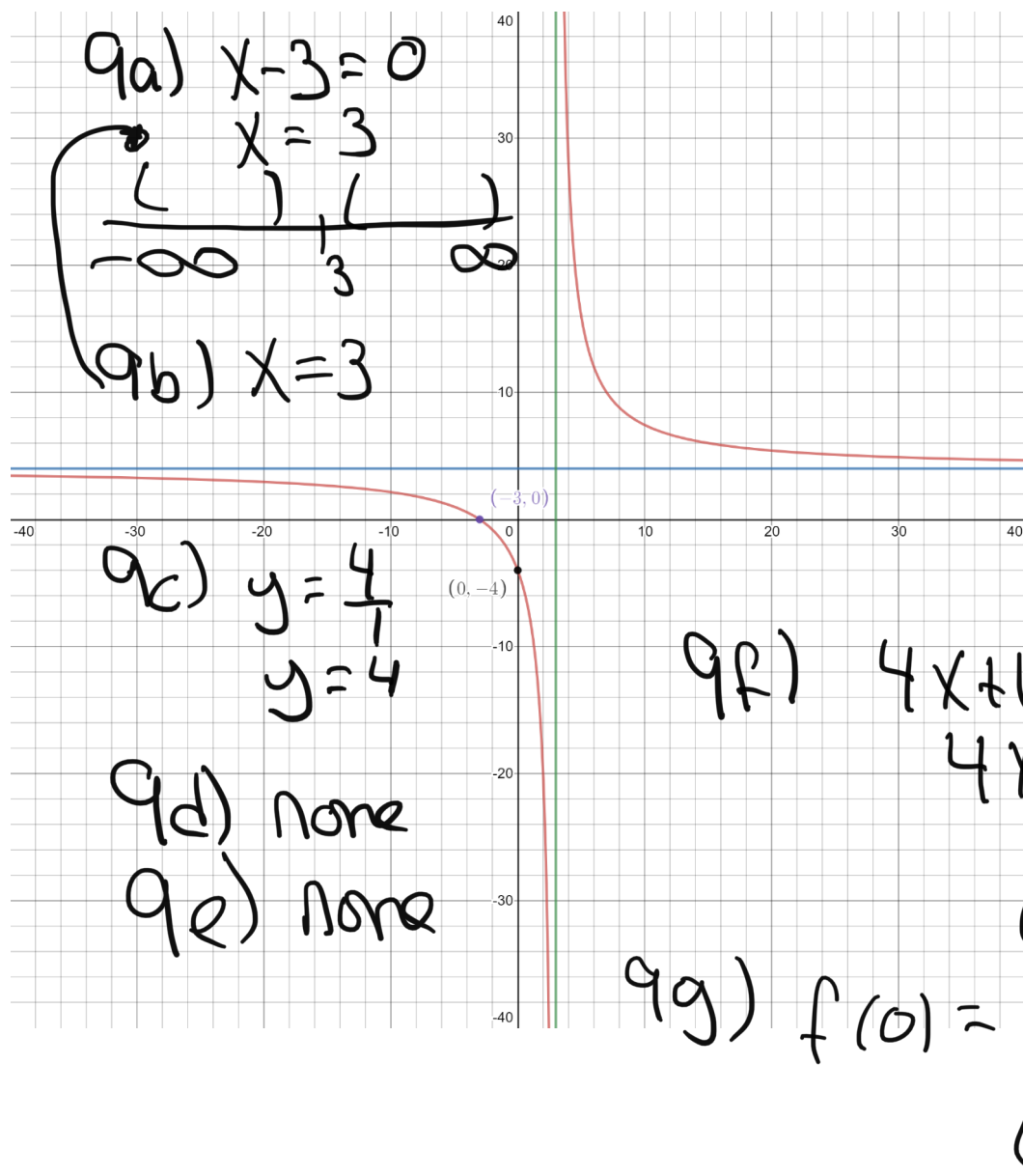
$\left(-2, -\dfrac{1}{10}\right)$

$(-2, -0.1)$

#17 - 28

For each problem find the following:
a) the domain of $f(x)$ written in interval notation $(-\infty, 3) \cup (3, \infty)$
b) the equation of the vertical asymptote (write none if there is no vertical asymptote) $x = 3$
c) the equation of the horizontal asymptote (write none if there is no horizontal asymptote) $y = 4$
d) the equation of the slant asymptote (write none if there is no slant asymptote) NONE
e) write the coordinates of any "hole" (write none if there is no hole) NONE
f) x- intercept(s) if any $(-3,0)$
g) y-intercept(s) if any $(0, -4)$
h) Sketch a graph of the function

17) $f(x) = \dfrac{4x+12}{x-3}$

9a) $X-3=0$
$X = 3$

9b) $X = 3$

9c) $y = \dfrac{4}{1}$
$y = 4$

9d) none

9e) none

9f) $4X+12 = 0$
$4X = -12$
$X = -3$
$(-3, 0)$

9g) $f(0) = \dfrac{4(0)+12}{0-3} = \dfrac{12}{-3}$
$(0, -4)$

For each problem find the following:
a) the domain of $f(x)$ written in interval notation $(-\infty, -6) \cup (-6, \infty)$
b) the equation of the vertical asymptote (write none if there is no vertical asymptote) $x = -6$
c) the equation of the horizontal asymptote (write none if there is no horizontal asymptote) $y = 3$
d) the equation of the slant asymptote (write none if there is no slant asymptote) NONE
e) write the coordinates of any "hole" (write none if there is no hole) NONE
f) x- intercept(s) if any $(4,0)$
g) y-intercept(s) if any $(0, -2)$
h) Sketch a graph of the function

19) $f(x) = \dfrac{6x-24}{2x+12}$

(4, 0)

(0, -2)
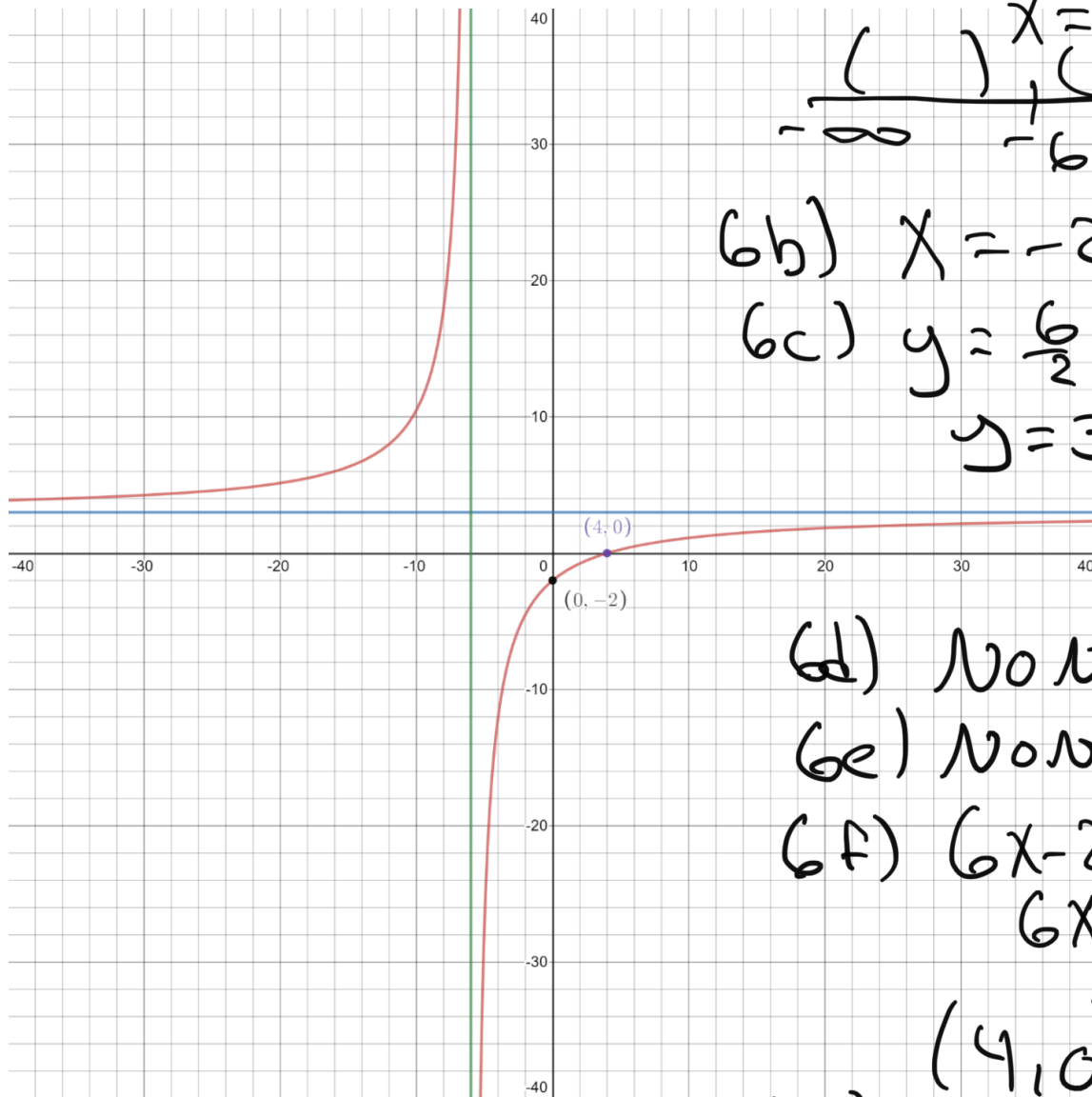
6a) $2x+12 = 0$
$2x = -12$
$x = -6$

$$\frac{(\quad)\,,(\quad\quad)}{-\infty \qquad -6 \qquad \infty}$$

6b) $x = -2$

6c) $y = \dfrac{6}{2}$
$y = 3$

6d) NONE

6e) NONE

6f) $6x - 24 = 0$
$6x = 24$
$x = 4$
$(4, 0)$

6g) $f(0) = \dfrac{6(0)-24}{2(0)+12}$
$= -2$
$(0, -2)$

For each problem find the following:
a) the domain of $f(x)$ written in interval notation $(-\infty, -7) \cup (-7, 1) \cup (1, \infty)$
b) the equation of the vertical asymptote (write none if there is no vertical asymptote)
$x = -7 \quad x = 1$
c) the equation of the horizontal asymptote (write none if there is no horizontal asymptote) $y = 0$
d) the equation of the slant asymptote (write none if there is no slant asymptote) NONE
e) write the coordinates of any "hole" (write none if there is no hole) NONE
f) x- intercept(s) if any $(14,0)$
g) y-intercept(s) if any $(0,2)$
h) Sketch a graph of the function

21) $f(x) = \dfrac{x-14}{x^2+6x-7}$
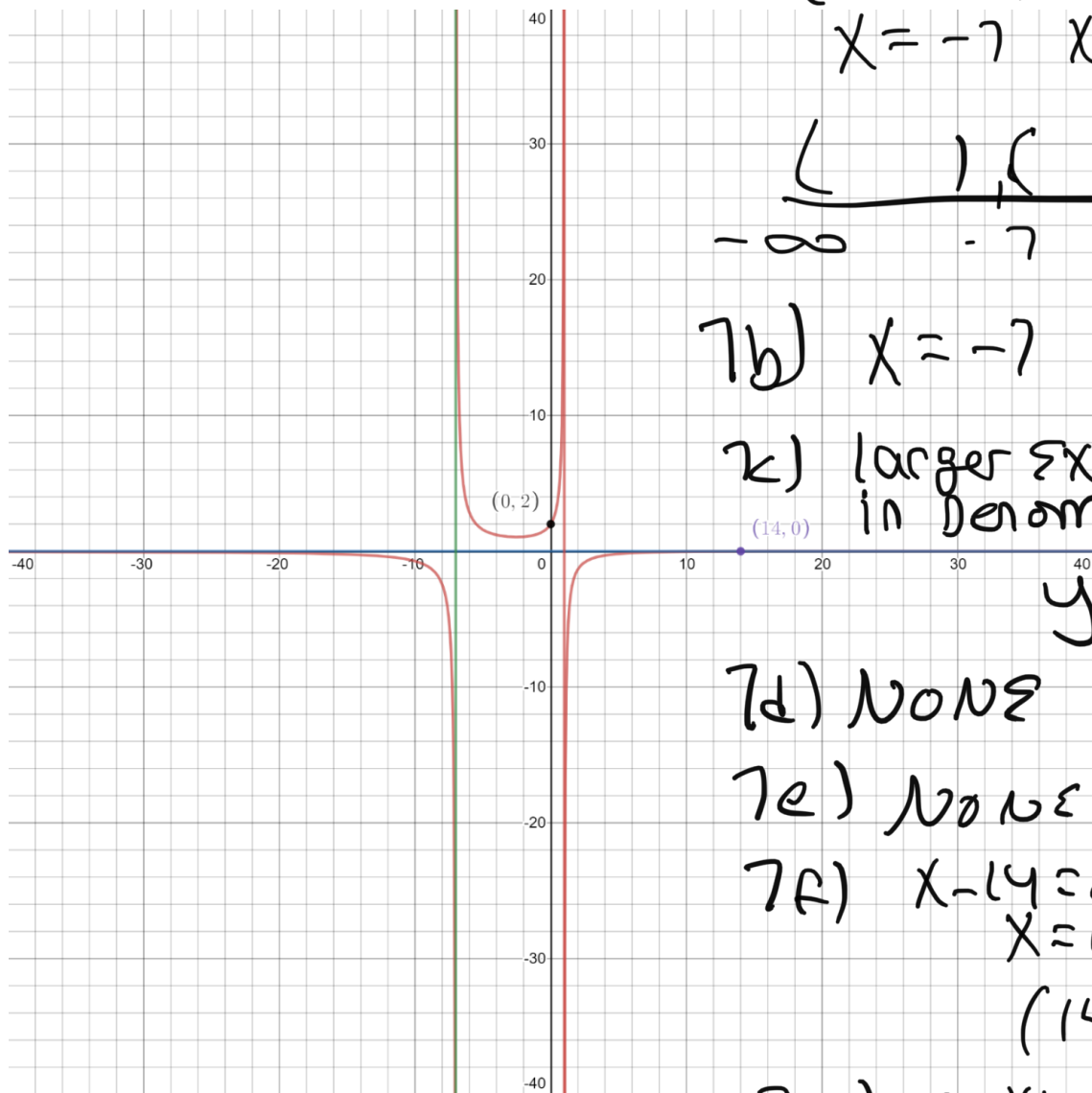
7a) $X^2 + 6X - 7 = 0$
$(X+7)(X-1) = 0$
$X = -7 \quad X = 1$

$\dfrac{(\quad)(\quad)(\quad)}{-\infty \qquad -7 \qquad 1 \qquad \infty}$

7b) $X = -7 \quad X = 1$

7c) larger exponent in Denom
$y = 0$
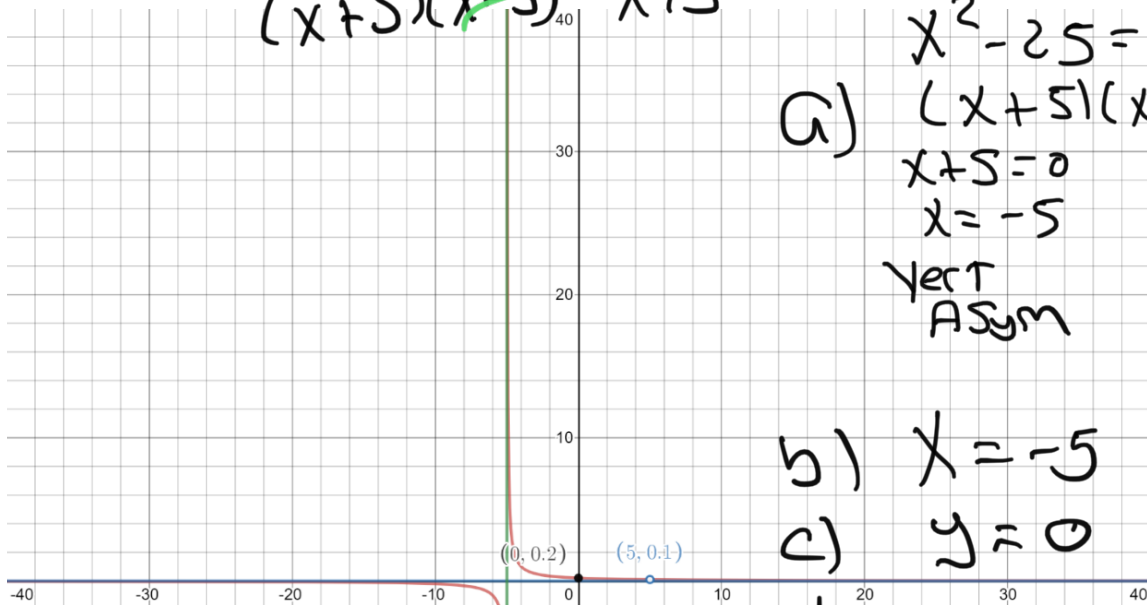
7d) NONE

7e) NONE

7f) $X - 14 = 0$
$X = 14$
$(14, 0)$

7g) $\dfrac{0-14}{0^2+6(0)-7} = 2$
$(0, 2)$

For each problem find the following:
a) the domain of $f(x)$ written in interval notation $(-\infty, -5) \cup (-5,5) \cup (5,\infty)$
b) the equation of the vertical asymptote (write none if there is no vertical asymptote) $x = -5$
c) the equation of the horizontal asymptote (write none if there is no horizontal asymptote) $y = 0$
d) the equation of the slant asymptote (write none if there is no slant asymptote) NONE
e) write the coordinates of any "hole" (write none if there is no hole) $\left(5, \frac{1}{10}\right)$
f) x- intercept(s) if any NONE
g) y-intercept(s) if any $\left(0, \frac{1}{5}\right)$.
h) Sketch a graph of the function

23) $f(x) = \dfrac{x-5}{x^2-25} = \dfrac{x-5}{(x+5)(x-5)} = \dfrac{1}{x+5}$ hole

$(0, 0.2)$ $(5, 0.1)$

$x^2 - 25 = 0$

a) $(x+5)(x-5) = 0$
$x+5 = 0 \qquad x-5 = 0$
$x = -5 \qquad x = 5$
VerT $\qquad$ hole
ASym

b) $x = -5$

c) $y = 0$
larger Exponent in denominator

d) none

e) $\dfrac{1}{x+5}$ Let $x = 5$
$\dfrac{1}{5+5} = \frac{1}{10}$
$\left(5, \frac{1}{10}\right)$

f) $\dfrac{1}{x+5}$
$1 \ne 0$ no Solution
NO X-INT

g) $f(0) = \dfrac{0-5}{0^2-25} = \dfrac{-5}{-25} = \frac{1}{5}$
$\left(0, \frac{1}{5}\right)$

For each problem find the following:
a) the domain of $f(x)$ written in interval notation $(-\infty, 4) \cup (4, \infty)$
b) the equation of the vertical asymptote (write none if there is no vertical asymptote) $x = 4$
c) the equation of the horizontal asymptote (write none if there is no horizontal asymptote) NONE
d) the equation of the slant asymptote (write none if there is no slant asymptote) $y = x + 10$
e) write the coordinates of any "hole" (write none if there is no hole) NONE
f) x- intercept(s) if any $(-8,0)$ $(2,0)$
g) y-intercept(s) if any $(0,4)$
h) Sketch a graph of the function

25) $f(x) = \dfrac{x^2 + 6x - 16}{x - 4}$
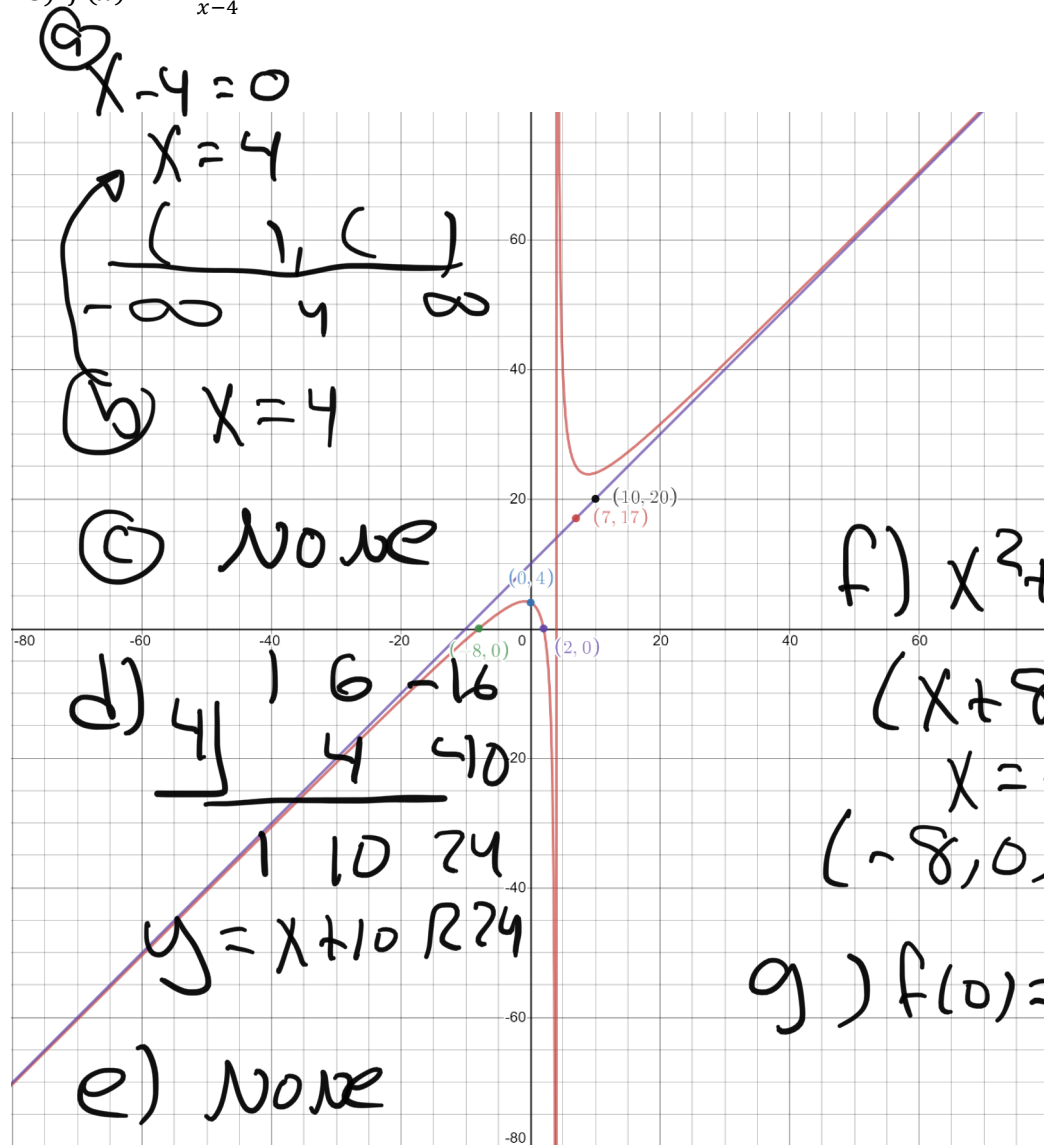
a) $x - 4 = 0$
$x = 4$

b) $x = 4$

c) None

d)
$$\begin{array}{r} x + 10 \\ x - 4 \overline{)\ x^2 + 6x - 16} \end{array}$$
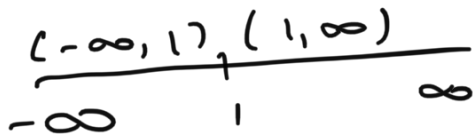
$y = x + 10 \ R\ 24$

e) None

f) $x^2 + 6x - 16 = 0$
$(x + 8)(x - 2) = 0$
$x = -8, 2$
$(-8, 0) \quad (2, 0)$

g) $f(0) = \dfrac{0^2 + 6(0) - 16}{0 - 4}$
$= \dfrac{-16}{-4}$
$= 4$
$(0, 4)$

27a) $X - 1 = 0$
$X = 1$

$(-\infty, 1) \cup (1, \infty)$

$$\frac{(-\infty, 1), (1, \infty)}{-\infty \quad | \quad 1 \quad | \quad \infty}$$

27b) $X = 1$

27c) NONE

27d)
$$1 \begin{array}{|rrr} 1 & 3 & -18 \\ & 1 & 4 \\ \hline 1 & 4 & -14 \end{array}$$

$y = X + 4$

$y = 1X + 4 \ R -14$

27e) none

27f) $X^2 + 3X - 18 = 0$
$(X + 6)(X - 3) = 0$
$X = -6 \quad X = 3$

$(-6, 0) \quad (3, 0)$

27g) $f(0) = \frac{0^2 + 3(0) - 18}{0 - 1} = \frac{-18}{-1} = 18$

$(0, 18)$